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# MARYLAND DEPARTMENT OF TRANPORTATION STATE HIGHWAY ADMINISTRATION

# **RESEARCH REPORT**

# STANDARDIZING LIGHTWEIGHT DEFLECTOMETER MODULUS MEASUREMENTS FOR COMPACTION QUALITY ASSURANCE

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> > APPENDICES

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#### APPENDIX 1. OTHER DOTS' FINDINGS

#### 1.1. Minnesota DOT

The MNDOT specification for using LWD for QA of geomaterials suggests defining the target LWD deflection based on the grading number (GN) (see Equation 1-1 below) and field moisture content for granular materials and on the plastic limit and field moisture content for fine-grained soils. The MNDOT specification requires testing immediately after compaction. Table 1-1 and Table 1-2 provide the target LWD deflection and modulus for granular materials and fine soils, respectively. The specification is provided in the following link: <a href="http://www.dot.state.mn.us/materials/gblwd.html">http://www.dot.state.mn.us/materials/gblwd.html</a>

$$GN(\% \text{ passing}) = \frac{25mm + 19mm + 9.5mm + 4.75mm + 2.00mm + 425\mu m + 75\mu m}{100}$$
 Equation 1-1

		Target LWD N	Target LWD	
GN		Keros/Dynatest	Zorn	Deflection
	%	MPa	MPa	mm
	5 -7	120	80	0.38
3.1 –	7-9	100	67	0.45
3.5	9 - 11	75	50	0.60
	5 -7	120	80	0.38
3.6 -	7-9	80	53	0.56
4.0	9 - 11	63	42	0.71
	5 -7	92	62	0.49
4.1 –	7-9	71	47	0.64
4.5	9 - 11	57	38	0.79
	5 -7	80	53	0.56
4.6 -	7-9	63	42	0.71
5.0	9 - 11	52	35	0.86
	5 -7	71	47	0.64
5.1 –	7-9	57	38	0.79
5.5	9 - 11	48	32	0.94
	5 -7	63	42	0.71
5.6 -	7-9	50	33	0.90
6.0	9 - 11	43	29	1.05

Table 1-1. LWD Target Values for Granular Material (Siekmeier et al. 2009)

Plastic	Estimated Optimum	Field Moisture as a % of Optimum	Zorn Deflec at Field Mo	Zorn Deflection Target at Field Moisture		
Limit	Moisture	Moisture	Minimum	Maximum		
%	%	%	mm	mm		
		70 - 74	0.5	1.1		
		75 - 79	0.6	1.2		
Non-	10.14	80 - 84	0.7	1.3		
plastic	10-14	85 - 89	0.8	1.4		
		90 - 94	1.0	1.6		
		70 - 74	0.5	1.1		
		75 - 79	0.6	1.2		
15 10	10 14	80 - 84	0.7	1.3		
13 - 19	10 - 14	85 - 89	0.8	1.4		
		90 - 94	1.0	1.6		
		70 - 74	0.8	1.4		
		75 - 79	0.9	1.6		
20 24	15 - 19	80 - 84	1.0	1.7		
20 - 24		85 - 89	1.2	1.9		
		90 - 94	1.4	2.1		
		70 - 74	1.0	1.7		
		75 - 79	1.2	1.9		
25 20	20 24	80 - 84	1.4	2.1		
23 - 29	20-24	85 - 89	1.6	2.3		
		90 - 94	1.8	2.6		
		70 - 74	1.3	2.0		
		75 - 79	1.5	2.2		
30 24	25 20	80 - 84	1.7	2.4		
30 - 34	23 - 29	85 - 89	1.9	2.7		
		90 - 94	2.2	3.0		

Table 1-2. LWD Target Values for Fine Grained Soil (Siekmeier et al. 2009)

White et al. (2009) in the study *Implementation of Intelligent Compaction Performance Based Specifications in Minnesota* developed and evaluated a new approach for LWD target value determination in the lab. Samples were compacted using a gyratory compaction (150 mm diameter x 150 mm high), since this is thought to simulate field compaction characteristics better than impact/Proctor compaction method. Samples were prepared using both granular and nongranular soil at various moisture and dry unit weights. LWD testing using a Zorn LWD with a 100-mm plate diameter was performed on the specimens while in the gyratory mold. The results were used to evaluate the moisture content-dry unit weight-LWD modulus relationship. Target values were then calculated based on reaching a 95% to 110% standard Proctor maximum density on the contour plots of multiple regression relationships.

Additionally, acceptance specification for the field projects in this study required constructing a control strip for each type of material used for grading. Optimum compaction is reached when

additional compactor passes do not result in significant increase in stiffness. The acceptable moisture content range was set between 65% to 95% of standard Proctor optimum moisture content. The LWD target value was defined as the average of three LWD tests performed on the control strip.

Overall, the LWD on gyratory mold method's target values were within the range of MNDOT suggested target modulus (Figure 1-1). Additional research was recommended to: (1) better simulate the field boundary conditions, (2) evaluate this mold method for a variety of soil types, and (3) obtain a cost-efficient test set up.



Figure 1-1. Comparison between LWD on gyratory mold target values, MNDOT target modulus, and in-situ LWD values (from White et. al, 2009)

#### 1.2. Indiana DOT

The INDOT also developed a test method for *Field Determination of Deflection Using Light Weight Deflectometer (ITM No. 508-12T)*. The INDOT specification can be used on granular soils, coarse aggregates and chemically modified soils. Specification link: <u>http://www.in.gov/indot/div/mt/itm/pubs/508\_testing.pdf</u> The INDOT specification provides two options to determine the LWD target deflection value: (1) Control strip and (2) Comparison. In the first method, the specification requires: (a) construction of a control strip to meet the specified requirements; (b) LWD testing during the control strip construction at specified intervals; and (c) plotting the average LWD deflection against the roller pass count. The minimum deflection is set as the target value. The deflection measured in the LWD test on the compacted layer should not exceed 1.1 time the LWD target value.

In the second method, the target value is selected based on comparisons with Dynamic Penetration Index (DPI) for granular base or the INDOT-specified density method for finegrained soils. The LWD target value is defined as the average LWD deflection at which minimum DPI or density values pass their respective criteria.

# 1.3. Florida DOT

Glagola et al (2015) proposed a specification for measuring deflection using a portable impulse plate load device in the *Performance-Based Quality Assurance/Quality Control (QA/QC) Acceptance Procedures for In-Place Soil Testing Phase 3* study. The specification requires LWD testing to be performed concurrently with the nuclear density testing so that both the density and modulus requirements are satisfied. There are two methods of obtaining the target maximum allowable deflection using LWDs: (1) correlations between LWD measurements and laboratory resilient modulus values (Table 1-3 and Table 1-4) for stabilized subgrade and limerock base material, and (2) direct measurement on a test strip for other type of geomaterial.

The test strip is constructed using the contractor's material and roller compactor. Ten LWD deflection measurements are conducted after four passes of roller on vibratory mode. Then an additional pass is made and another round of ten LWD deflection measurements taken. If the difference between average deflections are less than 0.02 mm, then the LWD average deflection values after fifth pass will be the target maximum deflection, else this procedure should continue until the average difference is less than 0.02 mm.

In case the average deflection after LWD testing in the field is more than the maximum allowable deflection (i.e., failing compaction), a moisture test should be performed. LWD testing is then repeated after 24 hours at the same location and if the deflections then are equal to or less than the maximum allowable deflection, then the compaction will be accepted.

It is recommended to conduct moisture content testing in the field with a Calcium Carbide gas pressure moisture tester. For lime rock, cemented coquina, and shell-rock base material, moisture content is measured with a NDG per Florida's sampling and testing methods.

	Belov (95%	v Optimum I Confidence	Moisture Interval)	At Optimum Moisture (95% Confidence Interval)		
Accelerometer- type LWD Deflection (mm)	Target*	Lower CI	Upper CI	Target	Lower CI	Upper CI
Stabilized Subgrade	0.39	0.33	0.44	0.55	0.46	0.65

 Table 1-3. Maximum LWD deflection for Florida stablized subgrade

\*-1% to optimum.

#### Table 1-4. Maximum LWD deflection for Florida limerock base

	Below (95%	Optimum M Confidence	foisture Interval)	At Optimum Moisture (95% Confidence Interval)		
Accelerometer- type LWD Deflection (mm)	Target*	Lower CI	Upper CI	Target	Lower CI	Upper CI
Limerock Base	0.23	0.19	0.27	0.26	0.23	0.29

\*-2% to optimum.

### 1.4. Nebraska Department of Roads

Nebraska department of roads (NDOR) uses LWD deflection plus moisture testing for QA of embankment, subgrade, foundation course, granular fill, and granular backfills according to NDOR *Lightweight Deflectomer (LWD) Field Testing – Quick Reference Guide*. Two methods of LWD target value determination are provided: (1) Based on resilient modulus correlations to Nebraska group index (NGI), and (2) Direct measurement from a test strip or calibration area. In the first method, the specification requires the moisture content to be within the specification limit, then the LWD target deflection should pass the values in the Table 1-5, depending on the unbound material location (under concrete or asphalt pavement, and for the top 3 feet or below 3 feet under the pavement).

When the deflection data for a specific soil type is not available, a 200-foot test strip must be compacted in two 8-inch lifts. Moisture content testing is required to confirm it is within the acceptable limits. Three LWD tests are conducted after each pass of the roller until the average deflection for three consecutive passes does not change significantly with any additional pass (less than 10% change). This final deflection value becomes the LWD target value. The field LWD deflections after compaction must not surpass 1.1 time the LWD target value.

Further research is recommended to complete the NGI vs LWD deflections for all soil types and conditions. Another recommendation was to search for a reliable field moisture content measurement device.

Nebraska	Concrete Upper 3'	Concrete Below 3'	Asphalt Upper 3'	Asphalt Below 3'
Group	Max	Max	Max	Max
Index	Deflection	Deflection	Deflection	Deflection
	(mm)	(mm)	(mm)	(mm)
-2	0.5	0.5	0.5	0.5
-1	0.5	0.5	0.5	0.5
0	0.5	0.5	0.5	0.5
1	1	1.5	0.5	1.5
2	1	1.5	0.5	1.5
3	2	3	1	3
4	2	3	1	3
5	2	3	1	3
6	2	3	1	3
7	1.5	3	0.75	3
8	1.5	3	0.75	3
9	1.5	3	0.75	3
10	2	4	1	4
11	2	4	1	4
12	2	4	1	4
13	2	4	1	4
14	3	5	2	5
15	3	5	2	5
16	3	5	2	5
17	4	6	3	6
18	4	6	3	6
19	5	8	4	8
20	5	8	4	8
21	5	8	4	8
22	6	9	5	9
23	6	9	5	9
24	6	9	5	9

Table 1-5. NDOR max allowable LWD deflection values for different NGI

## 1.5. European Union

The European Union (EU) also has developed a specification for LWD implementation (*CEN ICS 93.020*). In this specification two parameters are determined through LWD testing: (1) LWD modulus, which the EU specification terms the dynamic modulus, calculated based on the Boussinesq equation; and (2) dynamic compactness ( $T_{rd}$ ), which shows the quality of the compaction.  $T_{rd}$  is computed as:

$$T_{rd} = T_{rE} \times T_{rw}$$

**Equation 1-3** 

where  $T_{rE}$  is site relative compaction at a given water content and  $T_{rw}$  is the moisture correction coefficient to adjust for differences between the measured moisture content and optimum moisture content.  $T_{rw}$  is calculated as:

$$T_{rw} = \frac{\rho_{di}}{\rho_{d\max}}$$

In which  $\rho_{dmax}$  is the maximum dry density value obtained in the modified Proctor test and  $\rho_{di}$  is the dry density value from compaction curve of the modified Proctor tests corresponding to the in situ moisture content.

The testing process to obtain  $T_{rw}$  involves six sequences of LWD testing, each consisting of three LWD drops (total of 18 drops) on the loose, non-compacted material at the site. The average deformation of the second sequence is used to determine the initial dynamic modulus. The final dynamic modulus is calculated from the sixth testing sequence. According to the standard, the work imparted on the material during the six LWD sequences is equivalent to that applied in the modified Proctor test.

The EU specification determines the relative compaction  $T_{rE}$  at the field moisture content from the compaction curve using the following equation:

$$\%T_{rE} = 100 - \Phi_0 \times D_m$$
 Equation 1-4

where  $\Phi_0$  is a linear coefficient calculated from the Proctor-test results, typically assumed to be 0.365 ± 0.025, and D<sub>m</sub> is the deformation index, calculated from sum of the elements of the data line formed from the difference of the subsequent deflections up to the drop.

A similar approach was pursued by MNDOT (White et al., 2009) to compact the loose material in the field with multiple LWD drops so that the measured LWD modulus would asymptotically determine the target modulus. However, the target obtained by this method was lower than the average modulus measured after roller compaction at about the same moisture content. This was hypothesized to be due to the higher (by a factor of 3 to 4) compaction stresses under the roller as compared to the imparted stress under the 200 mm LWD plate.

### 1.6. United Kingdom

The *UK specification* defines four foundation classes according to the long term in-service foundation surface modulus values, as follows:

- Class  $1 \ge 50$ MPa
- Class  $2 \ge 100$ MPa
- Class  $3 \ge 200$ MPa

ad

where  $\lambda$ = -0.36 and F<sub>env</sub> = the relationship proposed by Cary and Zapata (2010) simplified by replacing wPI with zero:

- Class $4 \ge 400$ MPa	L
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For construction QC, the target mean and minimum modulus values are specified for the four foundation classes as shown in Table 1-6. The moving mean of five consecutive in-situ foundation surface modulus measurements must equal or exceed the target mean foundation surface modulus. All individual in-situ foundation surface modulus measurements must equal or exceed the target minimum foundation surface modulus.

Long-Term In-Service	Class I	Class II	Class III	Class IV		
Long-Term m-Service	≥50	≥100	≥200	≥400		
	Ur	ibound	40	80		
Target Mean Modulus		Fast Curing	50	100	300	600
(1111 a)	Bound	Slow Curing	40	80	150	300
	Ur	bound	25	50		
Target Minimum Modulus	Bound	Fast Curing	25	50	150	300
(1111 a)		Slow Curing	25	50	75	150

Table 1-6. UK specification. Target pavement foundation surface modulus

# 1.7. NCHRP 10-84 Study

The NCHRP 10-84 (2014) study proposed a framework for modulus based specification using LWD. The suggested procedure is to calculate the LWD effective modulus ( $E_{eff}$ ) from the Boussinesq equation after conducting the test in the field. Then,  $E_{eff}$  is adjusted using the following formula:

$$E_{\textit{adj}} = E_{\textit{eff}} \times K_{\textit{lab-field}} \times K_{\textit{moist}}$$

 $K_{lab-field} = (F_{env})^{\lambda}$ 

## In this equation, K<sub>lab-field</sub> is an adjustment factor that accounts for differences between lab and

**Equation 1-5** 

### Equation 1-6

$$\log(F_{env}) = \left( -.40535 + \frac{1.20693}{1 + e^{\left( 0.68184 + 1.33194 \left( \frac{S - S_{opt}}{100} \right) \right)}} \right)$$

**Equation 1-7** 

In this equation,  $S_{opt}$  = degree of saturation at optimum moisture content and S = degree of saturation at compaction moisture content.

K<sub>moist</sub> is an adjustment factor for differences in the compaction and testing moisture contents:

$$K_{moist} = e^{\eta(w_c - w_T)}$$
 Equation 1-8

where  $\eta$ =0.18 for fine-grained soils and 1.19 for unbound aggregates, w<sub>T</sub> is moisture content at time of testing (in percent); and w<sub>C</sub> is moisture content at time of compaction (in percent).

The specification suggests direct measurement of the resilient modulus of the geomaterial per AASHTO T-307 for target modulus determination. The pavement section is then modeled in a nonlinear structural pavement algorithm that simulates the center deflection under the LWD load the Boussinesq equation and using that to calculate the E<sub>target</sub>.

The evaluation of the proposed specification showed that achieving adequate layer modulus is weakly associated with achieving density. Changes in the assumed Poisson's ratio will highly affect the acceptance rate. Different LWDs estimate different moduli at the same test spot, suggesting the necessity for LWD specific specification. NCHRP 10-84 also found that it is very important to consider the properties of the underlying layers in setting the LWD target values, which is in line with findings from Khosravifar et al. (2013). A common concern regarding the modulus-based devices was the variability of the measurements. Based on tests on 20 independent specimens, Nazarian et al. (2014) found that the repeatability of the LWD devices is more than 15% and their reproducibility is more than 12%. Above 70% of variability in the LWD devices measurements was attributed to the variation in the materials properties.

Nazarian et al. (2013) only performed small scale testing in which specimens were compacted in a 90-cm diameter PVC pipe to a thickness of less than 0.55 m; these are much different conditions than in the field. Grasmick (2013) performed LWD testing on a large free-standing soil box structure with inside dimensions of 2.4m x 2.4m x 1.2m constructed out of timber. One significant issue reported in this work was the reflection of the LWD induced waves off the wood/soil box.

The online survey circulated among state Departments of Transportation (DOTs) as part of the NCHRP 10-84 project revealed that, while DOTs are interested in implementing a practical modulus-based specification, the incorporation of laboratory resilient modulus tests is not desirable.

Finally, NCHRP 10-84 recommended modulus-based acceptance tests to be implemented in conjunction with moisture measurements. Ironically, the nuclear density gauge was determined as the most reasonable moisture measurement tool in this study.

## APPENDIX 2. EQUIPMENT MODELING

### 2.1. Force Versus Height Assumptions for Zorn LWD

The load of Zorn LWD at drops other than the full height was estimated based on a single DOF mechanical model assumption illustrated in Figure 2-1.

A linear relationship can be derived between the square root of the drop height and the impact force. Upon releasing the falling weight, the gravitational potential energy (E) stored in the falling weight transforms to kinetic energy and then to elastic potential energy in the spring when the mass hits the spring, as shown in the following equations:



Figure 2-1. Similified schematic of LWD- soil motion as a one DOF mass-spring system.

$\boldsymbol{E} = \boldsymbol{m}.\boldsymbol{g}.\boldsymbol{h} = \frac{1}{2}\boldsymbol{k}.\Delta x^2$	Equation 2-1
$F = k. \Delta x$	Equation 2-2
therefore:	
$F = \sqrt{2. m. g. h. k}$	Equation 2-3

**Error! Reference source not found.** was used to estimate the impact force of Zorn LWD at h eights other than the full height when necessary.

#### 2.2. LWD Testing on Four-Pint Steel Beam

The performance of the three LWD devices was examined with the beam verification tester (BVT) developed by Hoffman et al. (2004). The BVT is a simply supported steel beam under two symmetric loads, also known as a four-point beam. The schematic of the beam is shown below:



L2 was 10 cm while 2L1+L2 was varied from 40 cm to 70 cm in 10 cm intervals to produce a linear elastic structure with different stiffness values. The static stiffness of the beam ( $k_s$ ) was measured by applying a ramp load at a slow rate of 0.2 mm/sec using an Instron Machine (Figure 2-5). The  $k_s$  was subsequently compared to the peak stiffness derived from LWD testing on the beam ( $k_{p-LWD}$ ). Figure 2-3 shows the Dynatest LWD on the beam as an example.

Using the LWD test data, the beam stiffness was calculated through two methods:

- a) Peak stiffness (k<sub>p-LWD</sub>) based on the ratio of peak load (F<sub>peak</sub>) to peak deflection (d<sub>peak</sub>) as reported by LWD (k<sub>p-LWD</sub>= F<sub>peak</sub>/d<sub>peak</sub>).
- b) Static Stiffness (k<sub>s-LWD</sub>). Based on the frequency response function and assumption of a single-degree-of-freedom mechanical model.

The most important objective of the BVT tests were to (1) verify the calibration and reliability of the three test devices on a linear elastic structure with known stiffness properties; and (2) assess the necessity to perform a full frequency domain analysis of the load and deflection signals for future LWD testing on soil. This study was based on work performed by Hoffmann et al. (2004), who found significant systematic errors of up to 278% between the  $k_{p-LWD}$  calculated from a Prima-100 LWD and the true stiffness of the beam (k<sub>s</sub>).

Tests were performed using the 300-mm diameter plate and the drop heights listed in Table 2-2, Table 2-3, and Table 2-4 for the Olson, Zorn, and Dynatest LWD respectively. Figure 2-4 presents the  $k_{p-LWD}$  versus the beam span for the evaluated LWDs. Tests were highly repeatable with coefficients of variation (CV) of less than 3% for all devices. The  $k_{p-LWD}$  for all three devices was overall in the same ballpark of the true stiffness of the beam, contrary to the finding of Hoffmann et al. (2004).

For Zorn and Olson, the  $k_{p-LWD}$  results were in line with true  $k_s$  of the beam at beam spans of 60 and 70 cm. As the beam span decreased, the  $k_{p-LWD}$  underestimated the stiffness. In case of the Dynatest LWD,  $k_p$  was close to the true stiffness of the beam at beam spans 40 and 50 cm and only slightly underestimated at higher beam spans.

The underestimation of the beam stiffness by Olson and Zorn in this experiment could be justified by the fact that these two LWDs measure the deflection of the plate whereas the

Dynatest LWD measures the deflection of the beam in the center of the plate. The underestimation was more significant for Zorn, which could be attributed to the error from assuming the peak load and different plate thickness.

Moreover, the Zorn and Olson devices are measuring the deflections of the quarter load points. Under static loading, the load point deflections are expected to be less than the centerline deflections. This would result in even higher errors for the Zorn and Olson stiffness than given here. However, under dynamic conditions it is conceivable that the centerline deflections during the initial portion of the response (i.e., at peak load and deflection) may be less than the load point deflections because of inertial lag.

Another explanation is that the deformations of the bearing cylinders at the load points (especially indentation effects) are included in the Zorn and Olson's deflections but not for the Dynatest's. This would cause the Zorn and Olson devices to overestimate deflections and thus underestimate stiffness, consistent with results shown in Figure 2-4.

Table 2-1 provides the  $k_s$  measured by the Instron machine. Table 2-2, Table 2-3, and Table 2-4 present  $k_{p-LWD}$  and the  $k_{p-LWD}/k_s$  ratio for the Olson, Zorn, and Dynatest LWDs, respectively.



Figure 2-3. Dynatest LWD test performed on BVT



Table 2-1. True static stiffness of the 4-point beam as measured by Instron

Figure 2-4.  $k_p$  as a function of beam span for the all the evaluated devices.



Figure 2-5. Measurement of BVT static stiffness (k<sub>s</sub>) with the Instron machine.

Span	Drop height	Force	d	k <sub>p-LWD</sub>	k <sub>p-LWD</sub> /k <sub>s</sub>
[cm]	[cm]	[kN]	[mm]	[MN/m]	[-]
40	2.5	1.70	0.463	3.68	0.7
40	5.1	2.20	0.594	3.70	0.7
40	7.6	2.80	0.752	3.72	0.7
40	10.2	3.09	0.837	3.69	0.7
40	12.7	3.38	0.923	3.66	0.7
40	21.6	4.72	1.268	3.72	0.7
50	2.5	1.51	0.814	1.86	0.8
50	5.1	1.96	1.075	1.82	0.8
50	7.6	2.37	1.285	1.85	0.8
50	10.2	2.80	1.503	1.86	0.8
50	12.7	3.17	1.680	1.89	0.8
50	21.6	4.13	2.162	1.91	0.8
60	2.5	1.47	1.311	1.12	0.8
60	5.1	1.88	1.620	1.16	0.8
60	7.6	2.23	1.925	1.16	0.8
60	10.2	2.58	2.214	1.17	0.8
60	12.7	2.90	2.453	1.18	0.8
60	21.6	3.85	2.861	Deflection sensor overload	
70	2.5	1.45	1.705	0.85	0.9
70	5.1	1.81	2.108	0.86	0.9
70	7.6	2.22	2.489	0.89	1.0
70	10.2	2.48	2.818	0.88	1.0
70	12.7	2.77	2.837	0.98	1.1
70	21.6	3.65	2.899	Deflection sensor overload	

Table 2-2. Peak stiffness (k<sub>p</sub>) from Olson LWD testing on 4 point beam.

Span	Drop height	Force	d	k <sub>p-LWD</sub>	k <sub>p-LWD</sub> /k <sub>s</sub>
[cm]	[cm]	[kN]	[mm]	[MN/m]	[-]
40	2.54	1.32	0.537	2.47	0.5
40	5.08	1.87	0.707	2.65	0.5
40	7.62	2.29	0.818	2.80	0.6
40	10.16	2.65	0.922	2.87	0.6
40	12.7	2.96	1.011	2.93	0.6
40	31.75	4.68	1.482	3.16	0.6
40	72.4	7.07	2.028	3.46	0.7
50	2.54	1.32	0.918	1.44	0.6
50	5.08	1.87	1.215	1.54	0.6
50	7.62	2.29	1.436	1.60	0.7
50	10.16	2.65	1.616	1.64	0.7
50	12.7	2.96	1.804	1.64	0.7
50	31.75	4.68	2.666	1.76	0.7
50	72.4	7.07	3.792	1.85	0.8
60	31.75	4.68	4.011	1.17	0.8
60	72.4	7.07	3.253	1.16	0.8
70	2.54	1.32	1.683	0.79	0.9
70	5.08	1.87	2.179	0.86	0.9
70	7.62	2.29	2.643	0.87	1.0
70	10.16	2.65	3.049	0.87	1.0
70	12.7	2.96	3.288	0.90	1.0
70	31.75	4.68	5.228	0.90	1.0
70	72.4	7.07	8.406	0.83	0.9

Table 2-3. Peak stiffness (k<sub>p</sub>) from Zorn LWD testing on 4 point beam.

Span	Drop height	Force	d	k <sub>p-LWD</sub>	k <sub>p-LWD</sub> /k <sub>s</sub>
[cm]	[cm]	[kN]	[mm]	[MN/m]	[-]
40	2.54	0.78	0.150	5.25	1.0
40	5.08	1.15	0.229	5.00	1.0
40	7.62	1.58	0.303	5.20	1.0
40	10.16	1.95	0.403	4.82	1.0
40	12.7	2.27	0.503	4.51	0.9
40	17.78	2.86	0.687	4.16	0.8
50	2.54	0.80	0.315	2.53	1.0
50	5.08	1.17	0.490	2.39	1.0
50	7.62	1.64	0.666	2.45	1.0
50	10.16	1.97	0.863	2.28	0.9
50	12.7	2.28	1.037	2.20	0.9
50	17.78	2.77	1.362	2.04	0.8
60	2.54	0.82	0.586	1.40	1.0
60	5.08	1.22	0.929	1.31	0.9
60	7.62	1.51	1.222	1.24	0.9
60	10.16	1.82	1.589	1.15	0.8
60	12.7	2.08	1.889	1.10	0.8
70	2.54	0.75	0.923	0.81	0.9
70	5.08	1.11	1.616	0.69	0.8
70	7.62	1.41	1.986	0.71	0.8
70	10.16	1.65	2.439	0.68	0.7
70	12.7	1.97	2.825	0.70	0.8

Table 2-4. Peak stiffness  $(k_p)$  from Dynatest LWD testing on 4 point beam.

#### 2.2.1. Frequency domain analysis

To account for the transient load and displacement data recorded on the BVT, the response was analyzed in the frequency domain using the Fast Fourier transform (FFT) algorithm in MATLAB. The technique has already been used by several researchers e.g. Hoffmann (2004).

In a linear system with y(t) and u(t) as the output and input of the system, respectively, the transient response of the linear system can be analyzed by the Frequency Response Function (FRF) defined as the ratio of the Fourier transform of the output, Y(f) and the Fourier transform of the input, U(f):

$$FRF(f) = \frac{Y(f)}{U(f)}$$
 Equation 2-4

where *f* is the frequency in Hz.

Assuming a linear model, the FRF of the dynamic stiffness, K(f) and mobility, M(f) are defined as follows:

$$K(f) = \frac{F(f)}{X(f)}$$
 Equation 2-5

$$M(f) = \frac{X(f)}{F(f)}$$
 Equation 2-6

where F(f), X(f), and  $\dot{X}(f)$  are the respective Fourier transforms of the force f(t), the displacement x(t), and the velocity  $\dot{x}(t)$ .

The static stiffness, k<sub>s</sub>, of this linear model can therefore be computed as the magnitude of the dynamic stiffness, K(f), at zero frequency. The geophone or accelerometer measurements are not reliable at low frequencies and thus the static stiffness cannot directly be calculated from the measurements (Hoffmann, 2004). Alternatively, the experimental data corresponding to the FRF of the dynamic stiffness at higher frequencies can be extrapolated to zero frequency. To get a more accurate extrapolation to zero frequency, the single degree of freedom (SDOF) mass (m), spring (with stiffness k), and damper (with damping coefficient c) linear system was considered to explain the LWD-beam movement. The ordinary differential equation (ODE) of motion for the SDOF mass-spring-damper system is as follows:

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = f(t)$$
 Equation 2-7

where  $\mathbf{a}(t)$  is the acceleration, m is summation of the mass of the LWD, including the receptacle and any other part on top of the verification beam, c represents the damping of the beam, and stiffness k is equal to the static stiffness of the verification beam (k<sub>s</sub>).



Figure 2-6. SDOF analog for the BVT: (a) four-point beam testing configuration; (b) corresponding SDOF analog (after Hoffmann, 2004).

The K(f) and M(f) functions can be derived by taking the Fourier transform from Equation 2-7 in the time domain. The functions as shown in Equation 2-8 and Equation 2-9 for K(f) and M(f), respectively, can be used for curve fitting and extrapolation of the experimental data to zero frequency. Since the force and deflection data were readily available in the tested LWDs, the spectral analysis using the dynamic stiffness function, K(f), was utilized.

$$K(f) = k \left[ \left( 1 - \beta^2 \right) + 2i\xi\beta \right]$$
 Equation 2-8

$$M(f) = \frac{1}{k} \frac{2i\pi f}{(1-\beta^2) + 2i\xi\beta}$$
 Equation 2-9

Where 
$$\beta = \frac{f}{f_n}$$
 is the tuning ratio,  $\xi = \frac{c}{4\pi m f_n}$  is the damping ratio, and  $f_n = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$  is the undamped natural frequency.

To reduce the effects of experimental noise and minimize the errors, a spectral average technique was used. This technique was effectively used by Bendat and Piersol (2011) and Guzina and Osburn (2002) for back calculating the FWD modulus on layered systems and by Hoffmann (2004) for the spectral analysis of the Prima-100 LWD impact load on the BVT. In the averaging technique, the dynamic stiffness function is estimated as:

$$K(f) = \frac{G_{xf}(f)}{G_{xx}(f)}$$
 Equation 2-10

where  $G_{xf}(f)$  is the one-sided cross-spectral density function of the deflection and force records and  $G_{xx}(f)$  is the one-sided auto-spectral density function of the deflection record.  $G_{xf}(f)$  and  $G_{xx}(f)$  were calculated using the built-in functions in MATLAB. The spectral averaging approach also allows for assessing the quality of the measurements and validity of the linearity assumption via the coherence function:

$$\gamma^{2}(f) = \frac{\left|G_{xf}(f)\right|^{2}}{G_{ff}(f)G_{xx}(f)}$$

#### **Equation 2-11**

where  $G_{\rm ff}$  (f) is the one-sided auto-spectral density function of the force record. The coherence function ranges from 0 to 1, where a value of zero shows no correlation between the input and output and a value of 1 shows the system is perfectly linear and noise free. The spectral analysis was not possible for the Zorn LWD since the load is not measured and a fixed load is assumed. An example of the spectral analysis on the Dynatest and Olson LWDs is shown in Figure 2-7 and Figure 2-8, respectively. Figure 2-9 depicts the strong agreement of the  $k_{p-LWD}$  and  $k_s$  values from the spectral analysis for the Dynatest LWD.



Figure 2-7. Spectral analysis on Dyantest LWD



Figure 2-8. Spectral analysis on Olson LWD



Figure 2-9. ks from spectral analysis versus kp of conventional peak method

#### 2.2.2. Conclusion

The objective was to distinguish the inherent differences in the LWD devices. To accomplish this task, LWD testing was performed on a four-point linear elastic steel beam with known stiffness properties. The stiffness from LWD was measured from (1) conventional peak method ( $k_{peak}$ ), by normalizing the peak force by peak deflection; and (2) through frequency domain spectral analysis to find the static k ( $k_s$ ). The results showed that the LWDs provide a good estimate of the beam stiffness through peak method and spectral analysis can only slightly improve the results.

# APPENDIX 3. RESILIENT MODULUS TESTING

### **3.1. Test Pits Material Characteristics**

In this section, the properties of the materials used in the test pits are described. The ALF soil used in the study was excavated from a hill at TFHRC and the VA21a stone and HPC were donated by the Luck Stone Company. The Unified Soil Classification and Atterberg Limits (AASHTO T-89 and T-90) of the studied soils are summarized in Table 3-1.

Material	Soil	PL	LL	PI	<b>D</b> <sub>90</sub>	<b>D</b> <sub>60</sub>	<b>D</b> <sub>30</sub>	<b>D</b> <sub>20</sub>	<b>D</b> <sub>10</sub>	P <sub>200</sub>	Cu	Cc
	classification											
[-]	[-]	[%]	[%]	[%]						[%]		
VA21a	GW	-	-	-	13.8	6.6	1.7	0.8	0.3	3.3	25.1	1.7
ALF	SM	27	31	5		0.2				41.6		
HPC	CH-MH	38	65	27						81.7		

Table 3-1. Material description

Figure 3-1 shows the gradation of the materials. The gradations were obtained per AASHTO T-27 for VA21a and per AASHTO T-11 and T-27 (wet and dry sieve analysis) for the ALF and HPC soils. The gradations were monitored throughout all phases of the project to ensure uniformity.



Figure 3-1. Gradation of the materials used in this study

The moisture-density relationships were determined for all three materials. Table 3-2 presents the optimum moisture content (OMC), maximum dry density (MDD) and bulk specific gravity of the test material. Figure 3-2 presents the moisture- density curves for all soils.

Soil Type	AASHTO Procedure	Method	Compaction energy	MDD	OMC	Specific Gravity <sup>(1)</sup>	
[-]	[-]	[-]	[-]	kg/m <sup>3</sup> (pcf)	[%]	[-]	
VA21a	T-99	D	Standard	2307.7 (144.0)	5.0	2 77	
VA21a	T-180	D	Modified	2435.9 (152.0)	4.5	2.11	
ALF	T-99	C	Standard	1923.1 (120.0)	11.5		
ALF	-	C	Semi Modified <sup>(2)</sup>	2003.2 (125.0)	10.5	2.71	
ALF	T-180	C	Modified	2083.3 (130.0)	9.5		
HPC	T-99	Α	Standard	1522.4 (95.0)	24.0	2.66	

Table 3-2. OMC, MDD and Specific gravity of the test material

 $(\overline{1})$  Specific gravity test per AASHTO T-84 and T-85

(2) Customized compaction energy: 3 layers, 25 drops per layer using a 4.54 kg rammer and a 457-mm drop



Figure 3-2. Moisture-Density relationships for (A) VA21a, (B) ALF, and (C) HPC.

# 3.2. Soil Water Characteristic Curve (SWCC)

The SWCC is the constitutive relationship existing between the degree of saturation of a soil (S) or volumetric water content and the associated matric suction. The degree of saturation is expressed by:

$$S = \frac{V_w}{V_v}$$
 Equation 3-1

The SWCC is an important constitutive relationship for unsaturated soils. The typical shape of the soil-water characteristic curve is showed in Figure 3-3. The suction (usually in logarithmic scale) increases as the degree of saturation (S) decreases. SWCC consists of three different zones as shown in Figure 3-3.



Figure 3-3. Schematic of the 3 zones of SWCC (Fredlund and Xing, 1994)

In the first zone, known as the boundary effect zone, the suction is low and the soil is in a completely saturated condition. An increment of reduction in degree of saturation (S) does not generate a substantial increase in suction. This situation continues until some air bubbles start to appear in the soil skeleton. The suction associated with this point is known as air-entry value, indicated as  $(u_a-u_w)_b$  or  $\psi_b$ .

The second part of SWCC is called the transition zone. It starts at the air-entry value. In this zone, an increment of reduction in degree of saturation is associated with a considerable increase in suction.

In the third part of curve, called the residual zone, a very significant reduction in degree of saturation is required for any further increase of suction. The suction value at the start of the third zone is marked as  $\psi_r$  and the associated S is known as the residual degree of saturation (S<sub>r</sub>).

The SWCC is commonly formulated as a nonlinear sigmoid function using the four-parameter equation proposed by Fredlund and Xing (1994):

#### **Equation 3-2**



**Equation 3-3** 

where:  $\theta_w$  = volumetric water content,

h = matric suction in kPa,

 $a_{f}$  = fitting parameter, which is primarily a function of the air entry value of the soil,

 $b_f$  = fitting parameter, which is primarily a function of the rate of water extraction from the soil, once the air entry value has been exceeded,

 $c_f$  = fitting parameter, which is primarily a function of the residual water content,  $h_{rf}$  = fitting parameter, which is primarily a function of the suction at which residual water content occurs, and

C(h) = correction factor which is a function of matric suction.

While SWCC can be directly measured in the lab, there are several studies that predict the SWCC parameters based on the gradation and soil index properties. In this study, the empirical model in the Mechanistic-Empirical Pavement Design Guide (MEPDG) was used to obtain the fitting parameters of the Fredlund and Zing SWCC equation. These equations are as follows:

(i)  
if 
$$P_{200}PI > 0$$
  
 $a_f = 0.00364 (P_{200}PI)^{3.35} + 4 (P_{200}PI) + 11$ , kPa  
 $\frac{b_f}{c_f} = -2.313 (P_{200}PI)^{0.14} + 5$   
 $c_f = 0.0514 (P_{200}PI)^{0.465} + 0.5$   
 $\frac{h_r}{a_f} = 32.44e^{0.0186(P_{200}PI)}$ 

**Equation 3-4** 

(ii)  
if 
$$P_{200}PI = 0$$
  
 $a_f = 0.8627 (D_{60})^{-0.751}$ , kPa  
 $\overline{b}_f = 7.5$   
 $c_f = 0.1772 \ln (D_{60}) + 0.7734$   
 $\frac{h_r}{a_f} = \frac{1}{D_{60} + 9.7e^{-4}}$ 

**Equation 3-5** 

The predicted SWCC for the test soils is shown in Figure 3-4.



Figure 3-4. SWCC for the material used in this study

#### 3.3. Resilient Modulus Sample Preparation and Testing

The Resilient Modulus ( $M_R$ ) of unbound materials is determined by repeated load triaxial compression tests per the AASHTO T-307 in the laboratory. Table 3-3 summarizes the testing plan for the  $M_R$  tests performed in the lab.

Each cycle of the axial stress is a haversine shape pulse with the duration of 0.1 second and the rest period of 0.9 second. During the rest period, a contact stress equal to 10% of the maximum axial stress ( $\sigma_{max}$ ) is maintained. The cyclic stress ( $\sigma_{cyclic}$ ) is therefore equal to  $\sigma_{max}$ - $\sigma_{contact}$ = 90%  $\sigma_{max}$ .

 $M_R$  is defined as the ratio of the amplitude of the repeated axial cyclic stress ( $\sigma_{cyclic}$ ) to the amplitude of the resultant recoverable axial strain ( $\epsilon_r$ ). Figure 3-5 and Figure 3-6 show the terms of a load pulse in  $M_R$  test, and the stress- strain relation for a given cycle in the test, respectively.

The tests were performed using a 100 kN Servo Hydraulic Dynamic Universal Testing Machine (UTM-100) from IPC Global in the University of Maryland Pavement Materials Laboratory (Figure 3-7). The original 100 kN capacity load cell of the machine was replaced with a smaller and hence more delicate load cell with a 6 kN capacity. Two external linear variable differential transformers (LVDT)s were used to record the deformations under the cyclic dynamic haversine load.

The samples were prepared in molds with a height to diameter ratio of 2 using the Proctor hammer. These molds are taller than those used in a conventional Proctor compaction test (Figure 3-7). The number of layers and drops per layer were adjusted for the tall  $M_R$  molds to achieve the densities like that of the Proctor test. Table 3-4 lists the adjusted numbers of layers and drops per layer used in compaction of the  $M_R$  samples unless otherwise stated (i.e., for the case of less than standard compaction for ALF Pit 1 material).



\* The rest period will be 0.9 s for hydraulic loading devices

Figure 3-5. Resilient Modulus Terms: contact stress, cyclic axial stress (σ<sub>cyclic</sub>), and maximum resilient vertical stress (σ<sub>max</sub>) (AASHTO T-307)


Figure 3-6. Stress-strain relationship in M<sub>R</sub> test



Figure 3-7. UTM- 100 apparatus and sample

Soil Type	Target MC	Target DD	Mold diameter	Compaction energy	Condition	# of Replicate	Base or Subgrade Procedure
[-]	[%]	[kg/m <sup>3</sup> (pcf)]	[mm]	[-]	[-]	[-]	[-]
VA21a	4.5	2435 (152)	150	Modified	Optimum- Pit 2, Pit 3	2	Base
ALF	11.5	1922.2 (120.0)	100	Standard	Optimum	3	Subgrade+Base
ALF	15.3	1837.3 (114.7)	100	Standard	Pit 2	2	Subgrade+8 cycles of Base
ALF	10.0	1771.2 (110.6)	100	<standard<sup>1</standard<sup>	~Pit 1	2	Subgrade+Base
HPC	24.0	1521.8 (95.0)	100	Standard	Optimum	2	Subgrade+Base
HPC	29.0	1457.7 (91.0)	100	Standard	Pit 3	2	Subgrade+8 cycles of Base

Table 3-3. Testing plan for  $M_R$  testing according to AASHTO T-307

(1) 3 layers- 15 drops per layer

	Standa	rd (T-99)	Modified (T-180)		
Proctor Mol	d	100 mm	150 mm	100 mm	150 mm
		mold	mold	mold	mold
Weight of the hammer	[kg]	2.495	2.495	4.54	4.54
Height of drop	[m m]	305	305	457	457
Number of drops per layer	[-]	25	56	25	56
Number of layers	[-]	3	3	5	5
Diameter of mold	[mm]	101.6	152.4	101.6	152.4
Height of mold	[mm]	116.3	116.3	116.3	116.3
Volume of mold	[cm <sup>3</sup> ]	0.94	2.12	0.94	2.12
Compaction energy/volume	[kNm/m <sup>3</sup> ]	594	591	2698	2686
		Star	ndard	Mod	ified
${ m M_R}$ Mold		<b>Star</b> 100 mm	ndard 150 mm	<b>Mod</b> 100 mm	<b>ified</b> 150 mm
M <sub>R</sub> Mold		Star 100 mm mold	ndard 150 mm mold	Mod 100 mm mold	<b>ified</b> 150 mm mold
M <sub>R</sub> Mold Weight of the hammer	[kg]	<b>Star</b> 100 mm mold 2.495	ndard 150 mm mold 2.495	<b>Mod</b> 100 mm mold 4.54	<b>ified</b> 150 mm mold 4.54
M <sub>R</sub> Mold Weight of the hammer Height of drop	[kg] [mm]	<b>Star</b> 100 mm mold 2.495 305	ndard 150 mm mold 2.495 305	Mod 100 mm mold 4.54 457	<b>ified</b> 150 mm mold 4.54 457
M <sub>R</sub> Mold Weight of the hammer Height of drop Number of drops per layer	[kg] [mm] [-]	Star           100 mm           mold           2.495           305           26	ndard 150 mm mold 2.495 305 48	Mod 100 mm mold 4.54 457 31	ified 150 mm mold 4.54 457 59
M <sub>R</sub> Mold Weight of the hammer Height of drop Number of drops per layer Number of layers	[kg] [mm] [-] [-]	Star           100 mm           mold           2.495           305           26           5	ndard 150 mm mold 2.495 305 48 9	Mod 100 mm mold 4.54 457 31 7	ified 150 mm mold 4.54 457 59 12
M <sub>R</sub> Mold Weight of the hammer Height of drop Number of drops per layer Number of layers Diameter of mold	[kg] [mm] [-] [-] [mm]	Star           100 mm           mold           2.495           305           26           5           101.6	ndard 150 mm mold 2.495 305 48 9 152.4	Mod 100 mm mold 4.54 457 31 7 101.6	ified 150 mm mold 4.54 457 59 12 152.4
M <sub>R</sub> Mold Weight of the hammer Height of drop Number of drops per layer Number of layers Diameter of mold Height of mold	[kg] [mm] [-] [-] [mm] [mm]	Star           100 mm           mold           2.495           305           26           5           101.6           203.2	ndard 150 mm mold 2.495 305 48 9 152.4 295.02	Mod 100 mm mold 4.54 457 31 7 101.6 203.2	ified 150 mm mold 4.54 457 59 12 152.4 295.02
M <sub>R</sub> Mold Weight of the hammer Height of drop Number of drops per layer Number of layers Diameter of mold Height of mold Volume of mold	[kg] [mm] [-] [-] [mm] [mm] [cm <sup>3</sup> ]	Star           100 mm           mold           2.495           305           26           5           101.6           203.2           1.65	ndard 150 mm mold 2.495 305 48 9 152.4 295.02 5.38	Mod 100 mm mold 4.54 457 31 7 101.6 203.2 1.65	ified 150 mm mold 4.54 457 59 12 152.4 295.02 5.38

# Table 3-4. The mold dimensions, number of layers and drops per layer for Proctor molds and $M_R$ molds for standard and modified compaction energy

The load combinations are chosen based on the location of the material in the pavement structure—whether it is a subgrade or base, and Table 3-6 present the test sequences for subgrade and base soils, respectively.

Sequence No	Confining Pressure, $\sigma_3$		Max. Axial Stress, max		Cyclic Stress,		Constant Stress		No. of Load Applications
1.00.	110550	ii <b>c</b> , 03	51655	, Omax	Ocy	ciic	0.1	σmax	rippiloutions
-	kPa	psi	kPa	psi	kPa	psi	kPa	psi	-
0	41.4	6	27.6	4	24.8	3.6	2.8	0.4	500-1000
1	41.4	6	13.8	2	12.4	1.8	1.4	0.2	100
2	41.4	6	27.6	4	24.8	3.6	2.8	0.4	100
3	41.4	6	41.4	6	37.3	5.4	4.1	0.6	100
4	41.4	6	55.2	8	49.7	7.2	5.5	0.8	100
5	41.4	6	68.9	10	62.0	9.0	6.9	1.0	100
6	27.6	4	13.8	2	12.4	1.8	1.4	0.2	100
7	27.6	4	27.6	4	24.8	3.6	2.8	0.4	100
8	27.6	4	41.4	6	37.3	5.4	4.1	0.6	100
9	27.6	4	55.2	8	49.7	7.2	5.5	0.8	100
10	27.6	4	68.9	10	62.0	9.0	6.9	1.0	100
11	13.8	2	13.8	2	12.4	1.8	1.4	0.2	100
12	13.8	2	27.6	4	24.8	3.6	2.8	0.4	100
13	13.8	2	41.4	6	37.3	5.4	4.1	0.6	100
14	13.8	2	55.2	8	49.7	7.2	5.5	0.8	100
15	13.8	2	68.9	10	62.0	9.0	6.9	1.0	100

Table 3-5. Testing sequence for subgrade soils

~	~ ~		~						
Sequence	Confi	ining	Max.	Axıal	Cyclic	Stress,	Constant		No. of Load
No.	Pressu	re, $\sigma_3$	Stress	$\sigma_{max}$	σαν	σcyclic		ess,	Applications
				,	5		0.1	$\sigma_{max}$	
-	kPa	psi	kPa	psi	kPa	psi	kPa	psi	-
0	103.4	15	103.4	15	93.1	13.5	10.3	1.5	500-1000
1	20.7	3	20.7	3	18.6	2.7	2.1	0.3	100
2	20.7	3	41.4	6	37.3	5.4	4.1	0.6	100
3	20.7	3	62.1	9	55.9	8.1	6.2	0.9	100
4	34.5	5	34.5	5	31.0	4.5	3.5	0.5	100
5	34.5	5	68.9	10	62.0	9.0	6.9	1.0	100
6	34.5	5	103.4	15	93.1	13.5	10.3	1.5	100
7	68.9	10	68.9	10	62.0	9.0	6.9	1.0	100
8	68.9	10	137.9	20	124.1	18.0	13.8	2.0	100
9	68.9	10	206.8	30	186.1	27.0	20.7	3.0	100
10	103.4	15	68.9	10	62.0	9.0	6.9	1.0	100
11	103.4	15	103.4	15	93.1	13.5	10.3	1.5	100
12	103.4	15	206.8	30	186.1	27.0	20.7	3.0	100
13	137.9	20	103.4	15	93.1	13.5	10.3	1.5	100
14	137.9	20	137.9	20	124.1	18.0	13.8	2.0	100
15	137.9	20	275.8	40	248.2	36.0	27.6	4.0	100

Table 3-6. Testing sequence for base soils

The MEPDG universal constitutive model was fit to the experimental data.

$$M_R = k_1 \cdot p_a \left(\frac{\theta}{p_a}\right)^{k_2} \cdot \left(\frac{\tau_{oct}}{p_a} + 1\right)^{k_3}$$

**Equation 3-6** 

where:

$$\begin{split} &M_R = \text{resilient modulus;} \\ &\theta = \sigma_1 + \sigma_2 + \sigma_3 = \sigma_1 + 3 \cdot \sigma_3 = \text{bulk stress;} \\ &\tau_{oct} = \frac{1}{3} \cdot \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2} = \text{octahedral shear stress;} \\ &p_a = \text{atmospheric pressure used to normalize the equation;} \\ &k_1, k_2, k_3 = \text{regression constants determined from the laboratory tests.} \end{split}$$

The regression coefficient  $k_1$  in MEPDG model is a positive number that is directly proportional to the modulus. The coefficient  $k_2$  is a positive value and is known as the stress hardening term;

this is most significant in granular material. The coefficient  $k_3$  is a negative value, known as the stress softening term. The  $k_3$  coefficient is more significant in clay, showing a reduction of modulus with an increase of the octahedral shear stress.

Table 3-7, Table 3-8, and Table 3-9 summarize the average test results and the coefficients of the  $M_R$  universal constitutive model for the ALF, HPC, and VA21a soils, respectively. For individual results for each test specimen please refer to Khosravifar (2015).

		ODT		DU
Sample ID	[-]	OPT	Pit 2	~Pit I
Achieved MC	[%]	11.9%	14.6%	9.4%
Achieved DD	[pcf]	118.9	116.5	110.6
	[kg/m3]	1904.3	1867.0	1771.2
Ра	[kPa]	101.3	101.3	101.3
k1	[-]	1437.4	177.6	793.9
k2	[-]	0.429	0.485	0.601
k3	[-]	-3.717	0.000	-2.023
SSE	[MPa <sup>2</sup> ]	131.6	384.1	1635.8
Sqr(SSE)	[MPa]	11.5	19.60	40.44
$\mathbb{R}^2$	[MPa <sup>2</sup> ]	98.1%	58.7%	66.2%
R <sup>2</sup> _adj	[MPa <sup>2</sup> ]	97.6%	52.8%	61.4%
Max Sample-to-Sample COV	[%]	32.5%	27.9%	5.0%
of M <sub>R</sub> at different stress states				
Average Sample-to-Sample COV	[%]	16.2%	13.2%	1.8%
of M <sub>R</sub> at different stress states				

Table 3-7. ALF M<sub>R</sub> test results

Test Condition/ Material	[-]	OPT	Pit 3
Achieved MC	[%]	24.5	30.8
Achieved DD	[pcf]	93.6	89.4
	[kg/m3]	1499.2	1432.1
Ра	[kPa]	101.3	101.3
k1	[-]	888.8	583.4
k2	[-]	0.378	0.095
k3	[-]	-0.843	-1.789
SSE	[MPa <sup>2</sup> ]	2261.6	259.0
Sqr(SSE)	[MPa]	47.56	16.09
$\mathbb{R}^2$	[MPa <sup>2</sup> ]	26.8	81.0
R <sup>2</sup> _adj	[MPa <sup>2</sup> ]	16.4	78.3
Max Sample-to-Sample COV	[%]	6.4	18.7
of M <sub>R</sub> at different stress states			
Average Sample-to-Sample COV	[%]	2.4	5.3
of M <sub>R</sub> at different stress states			

Table 3-8. HPC M<sub>R</sub> test results

Table 3-9. VA21a M<sub>R</sub> test results

Sample ID	[-]	VA21a_Ave OMC
Achieved MC	[%]	3.7
Achieved DD	[pcf]	153.4
	[kg/m3]	2458.0
Ра	[kPa]	101.3
k1	[-]	590.6
k2	[-]	0.824
k3	[-]	0.000
SSE	[MPa2]	2765.0
Sqr(SSE)	[MPa]	52.58
R2	[MPa2]	96.6
R2_adj	[MPa2]	95.7
Max Sample-to-Sample COV	[%]	47.9
of MR at different stress states		
Average Sample-to-Sample COV	[%]	17.6
of MR at different stress states		

#### 3.4. Evaluated Unsaturated Resilient Modulus Constitutive Models

Four types of subgrade and four types of granular base soil data from Andrei (2003) were used to evaluate the models. The soil type and description for each material is presented in

Table 3-10. All base materials and one of the subgrade soils were non-plastic. More information about the volumetric and mechanical properties of the soils can be found in Andrei (2003).

The soil water characteristic curves, which were key inputs to the evaluated models, were predicted from the gradation and soil indices using the Fredlund and Xing (1994) procedure as implemented in the MEPDG. The unconfined compression (Uc), which was input to one of the predictive models, was estimated from CBR values according to Black (1962).

For all the soils, the  $M_R$  tests were performed on specimens compacted with standard and modified Proctor energies at their corresponding optimum moisture content as well as above and below optimum. This resulted in a total of 6 scenarios for each soil.

	Soil Types	Description				
	Phoenix Valley Subgrade (PVSG)	Clayey Sand, SC				
ade	Yuma Sand Subgrade (YSSG)	Poorly Graded Gravel with Sand, GP, Non				
g		Plastic				
Sub	Flagstaff Clay Subgrade (FCSG)	Clayey Sand, SC				
	Sun City Subgrade (SCSG)	Clayey Sand, SC				
	Grey Mountain Base (GMAB2)	Well Graded Gravel with Sand, GW, Non Plastic				
	Salt River Base (SRAB2)	Poorly Graded Sand with Gravel, SP, Non				
e		Plastic				
Bas	Globe Base (GLAB2)	Poorly Graded Sand with Silt and Gravel, SP-				
, ,		SM, Non Plastic				
	Prescott Base (PRAB2)	Poorly Graded Sand with Silt and Gravel, SP-				
		SM, Non Plastic				

 Table 3-10- Soil Type and Description (From Andrei, 2003)

The 9 following models were selected for evaluation. The parameters of the following models were calibrated—except for M4 and M6 predictive models—based on the measured data at optimum moisture content and maximum dry density of the standard Proctor compaction test scenario. The models where subsequently used to predict the  $M_R$  at the other moisture-density conditions. The evaluated models are explained below:

M1 is the general nonlinear model implemented in the MEPDG and is a function of total bulk stresses. This model does not consider the effect of suction u.

M1: 
$$M_R = K_1 P_a \left(\frac{\sigma_{bulk}}{P_a}\right)^{K_2} \left(\frac{\tau_{oct}}{P_a} + 1\right)^{K_3}$$

**Equation 3-7** 

in which  $\sigma_{bulk} = \sigma_1 + \sigma_2 + \sigma_3 = \sigma_d + 3\sigma_c$ ,  $\sigma_1, \sigma_2, \sigma_3$  = three principal stresses,  $\sigma_d$  = deviatoric stress,  $\sigma_c$  = confining stress,  $\tau_{oct}$  = octahedral shear stress =  $\frac{\sqrt{3}}{2}\sigma_d$ , and the coefficients K<sub>1</sub>, K<sub>2</sub>, and K<sub>3</sub> are regression coefficients.

M2, the second evaluated model, is similar to M1, with the bulk effective stress ( $\sigma_{bulk} + 3u$ ) replacing  $\sigma_{bulk}$ . The reason for the multiplication of suction by 3 is that suction adds to each of the three principal effective stresses.

M2: 
$$M_R = K_1 P_a \left(\frac{\sigma_{bulk} + 3u}{P_a}\right)^{K_2} \left(\frac{\tau_{oct}}{P_a} + 1\right)^{K_3}$$
 Equation 3-8

M3, proposed by Liang et al (2008) adds a suction dependency term ( $\chi$ ) to the effective stress term. The suction dependency term was proposed by Khalili and Khabbaz (1998). In this model the suction term (u) is not multiplied by 3.

M3: 
$$M_R = K_1 P_a \left(\frac{\sigma_{bulk} + \chi u}{P_a}\right)^{K_2} \left(\frac{\tau_{oct}}{P_a} + 1\right)^{K_3}$$
 Equation 3-9  
 $\chi = \left(\frac{(u_a - u_w)_b}{u_a - u_w}\right)^{0.55} = \frac{u_{air-entry}}{u}$  Equation 3-10

The  $u_{air-entry}$  term is the suction at air entry level where air starts to enter the largest pores in the soil. The upper limit of  $\chi$  is equal to 1.

M4, proposed by Siekmeier (2011), has been found a suitable predictive model for subgrade and fine soils. The K<sub>1</sub>-K<sub>3</sub> coefficients are also predicted as a function of suction and volumetric moisture content from SWCC of the soils. The equations are as follows:

M4: 
$$M_R = K_1 P_a \left(\frac{\sigma_{bulk} + \theta_w f u}{P_a}\right)^{K_2} \left(\frac{\tau_{oct}}{P_a} + 1\right)^{K_3}$$
 Equation 3-11

in which  $K_1 = 800 \times \left(\frac{1}{5\theta_{sat}}\right)^{1.5} \times \left(\frac{1}{\log_{10}(u)}\right), K_2 = \log_{10}(u) - 1, K_3 = -8\theta_{sat},$ 

 $f = \theta_w^{10\theta_{sat}^3}, \theta_w$  = volumetric water content,  $\theta_{sat}$  = volumetric water content at saturation, and  $\chi = \theta_w f$ .

The  $\chi$  in M4 model is not bracketed by the upper bound of 1. The M4 predictive model was reevaluated as model M5, in which the K values were calibrated for each soil through nonlinear regression. The formula for *f* was kept the same.

Yan et al. (2013) proposed two predictive models for subgrade soils based on gene expression programming (GEP) to correlate  $M_R$  with routine properties of subgrade soils and state of stress. GEP I was computationally unstable for non-plastic soils and was found erroneous for plastic soil and has thus been excluded from the comparisons. The GEP II model, selected for evaluation as model M6—is displayed below:

#### **Equation 3-12**

$$\begin{aligned} \mathsf{M6:} \ & M_R = \mathrm{atan} \left\{ \gamma_d * \left[ \frac{\gamma_d - U_c}{P_I} \right] \right\} + \left\{ 2 * \left[ \frac{sqrt(P_I)}{P_{200}} \right] \right\} + \ & \sigma_d + \left\{ 2 * \sin\left[ \frac{\gamma_d * \exp\{\mathrm{atan}[\sin(P_{200})]\}}{P_{200}} \right] \right\} + \\ & (\sigma_d * \mathrm{atan}\{sqrt(P_{200}) - \left[ (\sigma_d * P_{200}) / \gamma_d \right] \} ) + \left\{ \mathrm{atan}[sqrt(U_c) - \gamma_d] + \mathrm{atan}(\gamma_d) \right\} \end{aligned}$$

in which  $U_c$  = unconfined compressive strength, PI = Plasticity Index, P200 = percentage passing the No. 200,  $\gamma_d$  = dry density, and  $\sigma_d$  = deviatoric stress.

Recently, Gu et al. (2014) evaluated a model proposed by Lytton (1995) and reported satisfactory predictions for base course aggregates. The model is:

$$M_R = K_1 P_a \left(\frac{\sigma_{bulk} - 3\theta_w f u}{P_a}\right)^{K_2} \left(\frac{\tau_{oct}}{P_a}\right)^{K_3}$$
Equation 3-13

The *f* parameter in this model is a function of  $\theta_a$  and  $\theta_u$ , which are the volumetric water contents of the soil at air entry and unsaturation, respectively. The parameter *f* is bracketed by the upper and lower bounds below:

$$f_{upper \ bound} = \left[ \left( \frac{\theta_a - \theta_w}{\theta_a - \theta_u} \right) + \frac{1}{\theta_w} \left( \frac{\theta_w - \theta_u}{\theta_a - \theta_u} \right) \right]$$
Equation 3-14
$$f_{lower \ bound} = \left[ \frac{1}{\left( \frac{\theta_a - \theta_w}{\theta_a - \theta_u} \right) + \theta_w \left( \frac{\theta_w - \theta_u}{\theta_a - \theta_u} \right)} \right]$$
Equation 3-15

Three *f* values were evaluated in the Lytton model to predict the resilient modulus, resulting in the following models.  $\chi = \theta_w f$  ranges from  $\theta_u$  to 1 and therefore is theoretically sound. **M7** based on  $f = \frac{f_{upper bound} + f_{lower bound}}{2}$ , **M8** based on  $f = f_{upper bound}$ , and **M9** based on  $f = f_{lower bound}$ . Figure 3-8 shows the bounds of the pore suction for Lytton (1995) model.



Figure 3-8. The bounds of pore suction for Lytton (1995).

#### 3.5. Comparison of Evaluated MR Predictive Models

Least squares analysis was applied to the measured data at optimum moisture content and maximum standard dry density for all models except for the M4 and M6 predictive models to find the best model. To evaluate the performance of the models, the root mean square error (RMSE) and average relative error (RE) of model prediction were calculated at each moisture condition (wet, dry, optimum), each compaction energy effort (standard and modified Proctor compaction effort), and overall for each soil and every model. RMSE, a measure of model accuracy, reflects both systematic and nonsystematic error variation and has the same units as M<sub>R</sub>, here reported in ksi. RE measures the systematic error or bias of the models. The definitions of these evaluation criteria are given as follows:

$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^{n} \left( (M_{R-predicted} - M_{R-measured})^2 \right)}$$
Equation 3-16

$$RE = \frac{\frac{1}{n} \sum_{i=1}^{n} (M_{R-predicted} - M_{R-measured})}{\frac{1}{n} \sum_{i=1}^{n} (M_{R-measured})} = \bar{e} / \overline{M_R}$$
 Equation 3-17

Figure 3-9 shows the distribution of RMSE of the evaluated models at different moisture and compaction energy conditions. As expected, all the models performed well at the optimum moisture and density, the condition at which was the model parameters were calibrated. Prediction errors were the highest at dry of optimum at both compaction efforts. Figure 3-10 presents the prediction bias of the models for the plastic and non-plastic soils. Overall, all models underpredicted at dry of optimum for non-plastic soils.

(a) Plastic soils



Figure 3-9. RMSE of evaluated models at different moisture and compaction energy condition for (a) Plastic, and (b) Non-plastic soils.



## Figure 3-10. Average RE of evaluated models at different moisture and compaction energy condition for (a) Plastic, and (b) Nonplastic soils.

The overall RMSE of prediction of the models per soil is shown in

Table 3-11. The shaded cells in the table present the most accurate model. Overall, model M8— Lytton (1995) with f<sub>upper bound</sub>—outperformed the other models for both plastic and neoplastic soils. The M2 model, which in fact is the effective stress model with f=1, performed very well for non-plastic soils but did not provide an acceptable prediction accuracy for plastic soils.

An example of the measured vs. predicted  $M_R$  by M2 for a plastic soil (PVSG) is shown in Figure 3-11 shows the RE for each model and soil type. Again, model M8 was overall the most consistent model for both plastic and non-plastic soils. Model M4 and M2, while outperformed in several soil types, were erroneous in several others and did not provide a consistent prediction over the range of the evaluated soils.

Figure 3-12 presents the RMSE and RE for model M8 at different moisture and compaction effort conditions. M8, although better than the other models, under predicted the moduli at dry of optimum and optimum moisture at the modified compaction condition for all soils and over predicted at wet of optimum for the standard and modified compaction conditions of the plastic soils.

Figure 3-13 shows the measured vs. predicted  $M_R$  for GMAB and PVSG for the M8 model. These two soils provided the most and least accurate predictions, respectively.

RMSE (ksi)	M1	M2	M3	M4	M5	M6	M7	M8	M9
1.PVSG	143.0	3452.0	129.8	292.4	139.8	141.9	99.9	90.3*	118.0
2.YSSG	88.0	67.9	87.1	268.4	87.9	96.8	87.5	87.4	87.5
3.FCSG	49.6	49.6	49.2	46.2	49.7	55.0	51.4	50.9	52.1
4.SCSG	107.7	2964.4	86.9	2138.2	102.7	110.4	57.8	60.4	64.9
5.GMAB	26.9	26.8	26.9	196.0	26.9	37.5	26.9	26.9	26.9
6.SRAB	48.3	44.5	48.0	81.6	48.3	50.3	48.3	48.1	48.4
7.GLAB	41.1	39.9	40.9	67.3	41.1	43.3	41.0	40.9	41.1
8.PRAB	47.0	46.0	46.9	208.7	47.0	47.7	47.0	47.0	47.1
Plastic	100.1	2155.3	88.6	825.6	97.4	102.4	69.7	67.2	78.3
NonPlastic	50.3	45.0	49.9	164.4	50.2	55.1	50.1	50.1	50.2
All	69.0	836.4	64.5	412.3	67.9	72.9	57.5	56.5	60.8

Table 3-11. Overall RMSE of the evaluated models for each soil.

\* The shaded cells show the model yielded the lowest RMSE of prediction for each Soil type.



Figure 3-11. M<sub>R-predicted</sub> VS. M<sub>R-measured</sub> - Model M2 for Soil PVSG.

RE, %	M1	M2	M3	M4	M5	M6	M7	M8	M9
1.PVSG	-83%	1310%	-76%	85%	-81%	-86%	-53%	-30%	-68%
2.YSSG	-55%	-43%	-55%	127%	-55%	-80%	-55%	-55%	-55%
3.FCSG	-58%	-66%	-65%	-51%	-60%	-75%	-70%	-69%	-71%
4.SCSG	-71%	1789%	-58%	1240%	-67%	-83%	-21%	-5%	-35%
5.GMAB	-23%	-23%	-23%	16%	-23%	-40%	-23%	-23%	-23%
6.SRAB	-36%	-34%	-36%	-28%	-36%	-41%	-36%	-36%	-36%
7.GLAB	-32%	-31%	-32%	-36%	-32%	-38%	-32%	-32%	-32%
8.PRAB	-35%	-34%	-35%	-50%	-35%	-41%	-35%	-35%	-35%

Table 3-12. Overall relative bias of the evaluated Models for each soil.

\* The shaded cells show the model yielded the lowest RMSE of prediction for each Soil type.



Figure 3-12. (a) RMSE and (b) RE at different moisture and compaction energy conditions for Model 8.



Figure 3-13. M<sub>R-predicted</sub> from model M8 VS. M<sub>R-measured</sub> for GMAB and PVSG soils.

Overall, model M8—the model proposed by Lytton (1995) using the upper bound of the suction resistance factor ( $\theta_w f$ )—was found to be the most accurate model over a wide range of fine and coarse and plastic and nonplastic soils used in pavements subgrades and bases. However, the

RMSE for all models were high and far from acceptable. Local biases existed in all the evaluated models. The models tended to systematically under predict the moduli at dry of optimum.

#### APPENDIX 4. FIELD VALIDATION TESTING

LWD test dates at each site is presented in Table 4-1. Also Table 4-2 summarizes the quantity of tests performed (NDG, LWD devices and oven drying moisture test -AASHTO T 265) at each test site.

Gradation test (AASHTO T27-11), Atterberg limits (AASHTO T89-13, and T90-00), and Specific Gravity (AASHTO T85-10, T84-10, and ASTM D854-14) testing was performed in the lab on the soil samples collected from each test site.

The basic soil parameters: uniformity coefficient (Cu), coefficient of gradation (Cc), the soil particle diameter corresponding to 30% finer in the particle-size distribution (D30), the diameter corresponding to 60% finer in the particle-size distribution (D60), and the diameter in the particle-size distribution curve corresponding to 10% finer also defined as the effective size (D10) are exhibited in Table 4-3 for each soil type at test sites evaluated in this project.

The gradation curve in plotted for each site in Section 4.1. Also, the aerial views of projects locations, and any important field condition or limitation description are presented in Section 4.1.

	State Location	Material Type	<b>Testing Date</b>	
1	Virginia	Phenix subgrade	7/30/15	
2		MD 5 waste contaminated (WC) embankment	8/5/15	
3		MD 5 subgrade	8/13/15	
4	Maryland	MD 337, deep GAB layer	8/14/15	
5		MD404 sand overlaying subgrade	10/15/15	
6		MD 404 subgrade	10/15/15	
7		MD 404 GAB	10/15/15	
8	New York	Embankment (local subgrade)	8/20/15	
9	I. diana	Cement modified subgrade	8/25/15	
10	Indiana	GAB	8/27/15	
11	Misser	Subgrade	-	
12	NIISSOUTI	GAB	8/26/15	
13	Florido	Subgrade	10/20/15	
14	FIORICA	Base	10/20/15	

#### Table 4-1. LWD field evaluation testing date

	Location	Soil Type	NDG	Zorn LWD	Dynatest LWD	Olson LWD	Oven Moisture Content
1	Virginia	Phenix subgrade	15	30	30	30	10
2		MD 5 waste contaminated embankment	20	60	60	0	40
3		MD 5 subgrade	30	90	90	90	60
4	Maryland	MD 337, deep GAB layer	2	60	60	60	20
5		MD404 sand overlaying subgrade	10	30	30	30	10
6		MD 404 subgrade	10				10
7		MD 404 GAB	10	30	30	30	10
8	New York	Embankment (local subgrade)	40	90	90	90	30
9	Indiana	Cement modified subgrade	0	60	60	60	0
1 0	mulana	GAB	0	30	30	30	11
1 1	Missouri	Subgrade	0	0	0	0	0
1 2	1v11550u11	GAB	30	90	90	90	30
1 3	Florida	Subgrade	10	30	30	0	10
1 4	riuliua	Base	20	60	60	0	21

Table 4-2. Quantities of field tests performed at each site with different devices

Location and	Daa	DIA	D	6	<u> </u>	Atterberg Limits			Specific
Soil Type	D30	D10	D60	Cc	Cu	LL	PL	PI	Gravity
Virginia, Phenix subgrade	0.21	0.09	0.48	1.01	5.49	-	-	non- plastic	2.67
MD 5, waste contaminated embankment	0.85	0.19	3.85	0.97	19.83	22.30	19.35	2.90	2.21
MD 5, subgrade	0.79	0.40	9.42	0.16	23.54	-	-	non- plastic	2.69
MD 337, deep GAB layer	0.88	0.10	3.29	2.41	33.89	-	-	non- plastic	2.71
MD 404, top subgrade	0.37	0.26	0.56	0.96	2.13	-	-	non- plastic	2.37
MD 404, local subgrade	0.57	0.30	1.42	0.76	4.70	-	-	non- plastic	2.45
MD 404, GAB	0.43	0.11	3.74	0.46	34.00	-	-	non- plastic	2.41
New York, embankment	0.24	0.14	0.36	1.19	2.56	-	-	non- plastic	2.68
Indiana, cement modified subgrade	1.90	0.19	6.00	3.17	31.58	26.90	17.74	9.16	2.55
Indiana, GAB	3.10	0.31	9.81	3.14	31.35	-	-	non- plastic	2.83
Missouri, subgrade	1.01	0.33	8.44	0.37	25.74	-	-	non- plastic	2.50
Missouri, GAB	2.09	0.34	8.15	1.59	24.17	-	-	non- plastic	2.62
Florida, Subgrade	0.20	0.15	0.26	1.01	1.80	-	-	non- plastic	2.57
Florida, Base	0.45	0.23	2.49	0.34	10.66	-	-	non- plastic	2.46

Table 4-3. Material characteristics for evaluated field soils

## 4.1. Projects Locations and Details

### 4.1.1. Virginia

**Project:** Tola road subgrade and base compaction **Address:** 1603 Tola Road, Phoenix Virginia 23959, GPS: 37.074161, -78.754267 **Remarks:** 

- Subgrade was compacted a week prior to the testing date. Subgrade surface was noticeably dry at the time of testing.
- Test site was contaminated with dried stiff clay from other parts of the roadway in some sections. Some clayey soil was carried by the trucks or compaction rollers from the other part of the road and compacted afterwards which resulted in higher variation of moduli in the field.
- Due to thunderstorm, the construction was canceled and no LWD testing was performed on the base layer.



Figure 4-1. Aerial view of the virgina Tola road evaluation site and test locations



Figure 4-2. Gradation Curve of Virginia site geomaterials

#### 4.1.2. Maryland

**Project:** MD 5 embankment construction and subgrade compaction on the embankment (Contract number PG494)

Address: MD 5, from Auth way to South of I-495/I-95 Remarks:

- A 2 feet deep embankment was compacted with a waste contaminated soil. The soil contained large pieces of recycled material such as glass, rubber and metal parts. Testing carried out every 1 hour on the 100 ft test section for 2 rounds.
- After a week, the subgrade material was placed over the dried embankment with a slope of about 3%.

Testing carried out every 1 hour on the 100 ft test section for 3 rounds.



Figure 4-3. Gradation Curve of MD5 site geomaterials



Figure 4-4. Aerial view of the MD5 field evaluation site

**Project:** MD 337 lane widening **Address:** 4701 Allentown road, Suitland, MD **Remarks:** 

- The local subgrade was a weak clay, so the lane was undercut for 3 ft. and replaced with GAB material. The initial 2 ft was compacted earlier.
- Testing was performed on top of a 6 inches GAB layer placed over the existing 2-ft layer.

- The 6-inch layer was compacted the day before testing at the end of the day. However, very little surface drying was observed on the top 0.5 inches.
- NDG was not available on site on the day of testing. Testing performed earlier on the 6-inch compacted layer reported PC of 98%.



Figure 4-5. Aerial view of the MD337 field evaluation site



Figure 4-6. Gradation Curve of MD337 site geomaterials

#### **Project:** MD 404 dualization (Contract number AW8965270) **Address:** 11419 Ridgely Rd, Ridgely, MD 21660 **Remarks:**

- A 5-ft embankment of local subgrade was compacted beforehand. Since the subgrade material was too wet at the time of placing the base, a 4-inch layer of uniform sand was compacted over the existing subgrade.
- Testing performed right after compaction on the sand layer.
- The GAB base layer was compacted in a layer of 6 to 8 inches.
- Testing performed right after GAB compaction.



Figure 4-7. Aerial view of the MD404 field evaluation site



Figure 4-8. Gradation Curve of MD404 site geomaterials

#### 4.1.3. New York

**Project:** Luther Forest Boulevard extension Address: 3 Hemphill Pl Ballston Spa, NY 12020 (project office) Remarks:

- The embankment constructed in layers of 8 inch to 1 ft thickness below the final grade.
- LWD and NDG testing performed on two lifts.
- The water content of the material was dryer than OMC at the time of compaction. Therefore, a truck was spraying water on top of the sand after placement. Spraying water was not possible on the test site as the LWD testing personnel were working.



Figure 4-9. Gradation Curve of New York site geomaterials



Figure 4-10. Aerial view of the New York Luther Forest Boulevard evaluation site

## 4.1.4. Missouri

**Project:** I-64 lane widening and road shoulder compaction **Address:** 601 Salt Mill Rd, Chesterfield, MO 63017 (project office) **Remarks:** 

- The concrete shoulder on the I-64 lane had been removed. The natural dirt (subgrade) below the shoulder was only compacted with 1 to 2 passes of roller compactor.
- A layer of about 4 inches of crushed lime stone (base) had been placed on top of the subgrade.
- Since the base layer had already been placed, we were unable to perform LWD testing on the subgrade. Soil samples were collected from the subgrade for lab testing.

• LWD and NDG testing on the base layer performed right after compaction in two rounds of one hour interval.



Figure 4-11. Aerial view of the Missouri I-64 evaluation site



Figure 4-12. Gradation Curve of Missouri site geomaterials

#### 4.1.5. Indiana

**Project:** I-65 to Worthsville road and Graham road construction (R-35187-A) **Address:** 1615 S Graham Road, Greenwood, IN 46143 **Remarks:** 

- Cement stabilized subgrade was compacted 5 days before testing. The layer's thickness was 14 inches total.
- The subgrade was cured and very stiff to excavate for moisture samples at the time of testing. Therefore, water content samples were obtained from the depth of 3 to 6 inches from the trench on the side on the road. The water content was measured using the Ohaus moisture analyzer on the site.
- INDOT does not use NDG tests for routine compaction quality control anymore. INDOT used Zorn LWD testing and proof rolling with a fully loaded tri-axle truck to evaluate the compaction.
- Base material was compacted on top of the cured cement stabilized subgrade (3 inches thickness).
- Testing on the base performed right after compaction on almost the same locations as the subgrade testing.



Figure 4-13. Aerial view of the Indiana Graham road evaluation site



Figure 4-14. Gradation Curve of Indiana site geomaterials

#### 4.1.6. Florida

**Project:** SR 23 construction, South Jacksonville, FL From SR 21 (Blanding Blvd.) to Duval county line **Address:** Branan Field Rd, Orange Park, FL 32065 **Remarks:** 

- Subgrade was compacted a week before LWD testing in the field. LWD and NDG testing on the subgrade performed right before base placement.
- Limerock base material was compacted to a thickness of 6 to 8 inches. LWD and NDG testing on the base performed right after compaction in two rounds of one-hour interval.



**Figure 4-15. Gradation Curve of Florida site geomaterials** 



Figure 4-16. Aerial view of the Florida SR23 field evaluation site

### 4.2. Summary of LWD Field Moduli, Measured Water Content, and NDG Results

Location and Soil Type	Round of Testing	Average MC [%] by NDG	Standard Deviation	%COV
Virginia, Phenix subgrade	1st	12.96	5.20	40.11
MD 5 waste	1st	9.94	2.60	26.19
contaminated embankment	2nd	9.46	2.18	23.02
	1st	4.32	0.33	7.57
MD 5 subgrade	2nd	3.88	0.35	9.00
	3rd	4.22	0.78	18.38
MD 404 subgrade	1st	6.01	0.78	13.04
MD 404 GAB	1st	2.81	0.28	10.13
Name Vaula	Lift 1, 1st	4.85	0.49	10.02
ambankmant (local	Lift 1, 2st	4.72	0.58	12.26
subgrade)	Lift 2, 1st	4.79	0.49	10.15
subgrade)	Lift 2, 2nd	4.68	0.59	12.69
Missouri, GAB	1st	4.53	0.90	19.78
	2nd	4.31	0.69	16.07
Florida, Subgrade	1st	8.11	1.01	12.42
Florida Base	1st	12.75	0.98	7.68
rioriua, dase	2nd	12.12	0.53	4.35

Table 4-4. Summary of field water content measured by NDG

Location and Soil Type	Round of Testing	Average MC [%] by Oven drying	Standard Deviation	%COV
Virginia, Phenix subgrade	1st	8.70	2.92	33.57
MD 5 waste	1st	11.30	3.42	30.28
contaminated embankment	2nd	10.38	2.39	23.05
	1st	4.34	0.35	7.98
MD 5 subgrade	2nd	3.37	0.40	11.76
	3rd	2.56	0.89	34.61
MD 337, deep GAB layer	1st	2.40	0.42	17.50
N V1-	Lift 1, 1st	6.24	0.63	10.12
ambankmont (local	Lift 1, 2st	N/A	N/A	N/A
enibalikillent (local	Lift 2, 1st	5.79	0.44	7.55
subgrade)	Lift 2, 2nd	6.00	0.47	7.85
Indiana, cement modified subgrade	1st	N/A	N/A	N/A
Indiana, GAB	1st	6.44	0.32	4.98
Missouri, GAB	1st	4.77	0.92	19.27
	2nd	4.63	0.60	12.91
Florida, Subgrade	1st	8.79	0.90	10.25
Florida Pasa	1st	12.95	0.64	4.91
riuliua, Dase	2nd	12.88	0.66	5.12

Table 4-5. Summary of field water content obtained by oven drying method

Location and Soil Type	Round of Testing	Average %PC	Standard Deviation	%COV
Virginia, Phenix subgrade	1st	96.8	4.525	4.673
MD 5 waste contaminated	1st	97.9	4.857	4.960
embankment	2nd	98.3	3.977	4.042
	1st	98.6	2.328	2.360
MD 5 subgrade	2nd	98.4	1.674	1.700
	3rd	98.8	1.527	1.545
MD 337, deep GAB layer	1st	98.0	N/A	N/A
MD 404 subgrade	1st	N/A	N/A	N/A
MD 404 GAB	1st	90.2	1.214	1.345
	Lift 1, 1st	84.8	1.565	1.845
New York, embankment	Lift 1, 2st	85.4	2.531	2.963
(local subgrade)	Lift 2, 1st	83.2	2.020	2.425
	Lift 2, 2nd	83.2	1.950	2.343
Indiana, cement modified subgrade	1st	N/A	N/A	N/A
Indiana, GAB	1st	N/A	N/A	N/A
Missouri CAD	1st	100.0	3.585	3.584
MISSOUII, GAB	2nd	99.6	3.339	3.354
Florida, Subgrade	1st	90.8	1.462	1.609
Elorida Daga	1st	102.7	1.617	1.574
Fiorida, Base	2nd	102.4	1.327	1.295

Table 4-6. Summary of Percent Compaction values measured by NDG in the field

Location and Soil Type	Round of Testing	Average E <sub>field</sub> [MPa] Olson LWD	Standard Deviation	%COV
Virginia, Phenix subgrade	1st	19.484	13.920	71.446
	1st	82.100	36.179	44.067
MD 5 subgrade	2nd	77.571	27.815	35.858
	3rd	72.237	22.395	31.003
MD 337, deep GAB layer	1st	68.752	7.705	11.207
MD 404 subgrade	1st	36.704	6.408	17.458
MD 404 GAB	1st	35.997	5.229	14.526
Now York on houling out	Lift 1, 1st	22.495	4.068	18.084
(local subgrada)	Lift 2, 1st	19.299	2.987	15.476
(local subgrade)	Lift 2, 2st	19.366	3.517	18.159
Indiana, cement modified subgrade	1st	101.530	45.238	44.556
Indiana, GAB	1st	82.826	27.852	33.627
Missouri CAR	1st	46.834	13.826	29.523
WIISSOUII, GAB	2nd	55.494	16.285	29.345

Table 4-7. Summary of Olson LWD moduli on the field sites
Location and Soil Type	Round of Testing	Average E <sub>field</sub> [MPa] Zorn LWD	Standard Deviation	%COV
Virginia, Phenix subgrade	1st	27.786	22.281	80.187
MD 5 waste	1st	10.401	4.019	38.641
contaminated embankment	2nd	11.983	5.542	46.248
	1st	65.954	25.801	39.119
MD 5 subgrade	2nd	62.536	24.884	39.792
	3rd	69.263	23.889	34.491
MD 337, deep GAB layer	1st	64.713	8.059	12.454
MD 404 subgrade	1st	33.404	8.752	26.199
MD 404 GAB	1st	35.122	5.519	15.714
	Lift 1, 1st	19.861	2.797	14.083
New York, embankment	Lift 1, 2st	22.338	2.752	12.321
(local subgrade)	Lift 2, 1st	19.096	3.595	18.828
	Lift 2, 2nd	19.499	3.776	19.364
Indiana, cement modified subgrade	1st	82.240	41.411	50.354
Indiana, GAB	1st	71.105	27.580	38.787
Missessi CAD	1st	39.209	11.040	28.156
WIISSOUII, OAB	2nd	46.455	12.508	26.925
Florida, Subgrade	1st	71.521	7.265	10.157
Florida Pasa	1st	66.411	10.155	15.292
FIOTUA, Dase	2nd	73.261	9.858	13.456

Table 4-8. Summary of Zorn LWD moduli on the field sites

Location and Soil Type	Round of Testing	Average E <sub>field</sub> [MPa] Dynatest LWD	Standard Deviation	%COV
Virginia, Phenix subgrade	1st	71.015	59.098	83.219
MD 5 waste	1st	11.161	5.901	52.872
contaminated embankment	2nd	16.506	9.199	55.732
	1st	117.981	94.610	80.190
MD 5 subgrade	2nd	129.931	85.468	65.779
	3rd	147.965	134.758	91.074
MD 337, deep GAB layer	1st	115.928	21.073	18.178
MD 404 subgrade	1st	44.716	13.765	30.783
MD 404 GAB	1st	45.572	9.733	21.357
New York, embankment	Lift 1, 1st	28.783	4.537	15.763
(local subgrade)	Lift 2, 1st	29.690	5.257	17.706
Indiana, cement modified subgrade	1st	355.935	338.148	95.003
Indiana, GAB	1st	152.329	130.487	85.661
Missouri CAD	1st	71.446	29.633	41.476
Missouri, GAB	2nd	86.247	37.080	42.993
Florida, Subgrade	1st	107.789	16.347	15.166
Florida Pasa	1st	95.327	26.166	27.449
FIOITUA, DASE	2nd	118.179	16.515	13.975

Table 4-9. Summary of Dynatest LWD moduli on the field sites

## 4.3. Developing Correlation Between Moduli at First and Second Half-Heights Drops

Figure 4.17, Figure 4-18, and Figure 4-19 present the moduli from second half-height drop versus moduli from first half-height drop for Zorn, Dynatest and Olson LWDs respectively. Correlation equation and coefficient of determination (R<sup>2</sup>) are shown for each round of testing (R) and compacted lift (L) at each test site.









Figure 4-18. Comparison of moduli at first half-height drop and moduli at second halfheight drop for Dynatest LWD.





Figure 4-19. Comparison of moduli at first half-height drop and moduli at second halfheight drop for Olson LWD.

## 4.4. Regression Analysis

Table 4-10 to Table 4-14 present the regression coefficient and statistics for evaluated soils in Section 4.6 final report. Target values were then calculated by inputting the field GWC and P/Pa using the coefficients below. Regression statistics tables present the square root of  $R^2$  (Multiple R), the  $R^2$  (R Square), the adjusted  $R^2$  used when more than on variable applied, the sample estimate of the standard deviation of the error as the standard error, and the number of observations used in the regression for each LWD.

MD 5 subgrade	Regression Coefficients		
	Zorn	Dynatest	Olson
	LWD	LWD	LWD
Intercept	203.96	73.17	210.15
GWC	-269.41	0.00	-219.99
GWC^2	135.45	-84.21	87.27
GWC^3	-29.80	44.30	-13.61
GWC^4	2.94	-7.81	0.72
GWC^5	-0.11	0.45	0.00
P/Pa	9.18	208.41	6.79
P/Pa^2	-0.87	-285.03	1.01
P/Pa^3	0.00	122.00	0.00

Table 4-10.	Regression	coefficients a	nd statistics	for MD 5	suborade
1 abic 7-10.	Regression	coefficients a	ind statistics		subgraue

MD 5 subgrade	<b>Regression Statistics</b>		
	Zorn		
	LWD	Dynatest LWD	Olson LWD
Multiple R	0.9489	0.7574	0.9866
R Square	0.9003	0.5736	0.9733
Adjusted R			
Square	0.8798	0.3925	0.9664
Standard Error	4.1649	36.5916	2.7220
Observations	42	30	30

NY			
Embankment	Regression	Coefficients	
	Zorn	Dynatest	Olson
	LWD	LWD	LWD
Intercept	53.15	1724.25	-407.05
GWC	-5.48	-57.80	0.00
GWC^2	-0.76	6.98	83.89
GWC^3	0.08	-0.28	-20.81
GWC^4	0.00	0.00	1.88
GWC^5	0.00	0.00	-0.06
P/Pa	20.01	-3476.59	12.57
P/Pa^2	-2.67	2669.25	-2.87
P/Pa^3	0.00	-676.92	0.37

Table 4-11. Regression coefficients and statistics for NY embankment soil

NY Embankment	Regression Statistics		
	Zorn		
	LWD	Dynatest LWD	Olson LWD
Multiple R	0.9362	0.9572	0.8105
R Square	0.8765	0.9163	0.6569
Adjusted R			
Square	0.8507	0.8777	0.5023
Standard Error	4.1907	4.9758	16.7743
Observations	30	20	30

 Table 4-12. Regression coefficients and statistics for MD 337 GAB

MD 337 GAB	Regression Coefficients		
	Zorn	Dynatest	Olson
	LWD	LWD	LWD
Intercept	34.72	8.63	343.18
GWC	0.00	0.00	-275.21
GWC^2	-16.42	1.22	79.59
GWC^3	3.52	-0.03	-7.47
GWC^4	0.00	0.00	0.00
GWC^5	0.00	0.00	0.00
P/Pa	48.03	102.28	-1.39
P/Pa^2	-13.27	-75.42	4.82
P/Pa^3	1.54	21.15	-0.33

MD 337 GAB	Regression Statistics		
	Zorn		
	LWD	Dynatest LWD	Olson LWD
Multiple R	0.9598	0.9597	0.9442
R Square	0.9212	0.9209	0.8915
Adjusted R			
Square	0.8050	0.8047	0.8532
Standard Error	4.6596	3.4683	6.7559
Observations	18	18	24

Table 4-13. Regression coefficients and statistics for MD 404 GAB

MD 404 GAB	Regression Coefficients		
	Zorn	Dynatest	Olson
	LWD	LWD	LWD
Intercept	41.66	2064.44	-337.98
GWC	-4.88	1192.86	189.31
GWC^2	0.19	-197.11	-30.35
GWC^3	0.00	10.78	1.60
GWC^4	0.00	0.00	0.00
GWC^5	0.00	0.00	0.00
P/Pa	40.57	-10163.89	-1.54
P/Pa^2	-6.97	7860.82	5.93
P/Pa^3	0.00	-2012.39	-1.35

MD 404 GAB	Regression Statistics		
	Zorn		
	LWD	Dynatest LWD	Olson LWD
Multiple R	0.9829	0.9284	0.5648
R Square	0.9660	0.8619	0.3190
Adjusted R			
Square	0.9556	0.7982	0.0921
Standard Error	2.2850	26.8883	8.0510
Observations	18	20	25

FL Base	Regression Coefficients		
	Zorn	Dynatest	
	LWD	LWD	
Intercept	460.80	-2829.00	
GWC	0.00	979.38	
GWC^2	-11.83	-84.64	
GWC^3	0.69	2.36	
GWC^4	0.00	0.00	
GWC^5	0.00	0.00	
P/Pa	102.10	-1866.55	
P/Pa^2	-31.29	1677.53	
P/Pa^3	0.00	-509.43	

Table 4-14. Regression coefficients and statistics for FL base material

FL Base	Regression Statistics	
	Zorn LWD	Dynatest LWD
Multiple R	0.9852	0.9433
R Square	0.9706	0.8899
Adjusted R		
Square	0.8589	0.8611
Standard Error	4.2806	32.7026
Observations	15	30